SAT Subject Physics Facts & Formulas

This document is a concise but comprehensive guide to the facts and formulas typically used in the material covered by the SAT Subject physics test. The test is designed to determine how well you have mastered the physics concepts taught in a typical one-year college-prep high school course.

This guide is mainly intended as a reference, as opposed to a full tutorial (which would probably be book-length), and so the explanatory material is pretty brief. You can use the guide as a simple formula reference, or as a quick review of the material that you've already studied elsewhere. Either way, good luck on your Subject Test!

Math Stuff

Although this guide is for the SAT Subject test in Physics, you'll need to know quite a bit of math. If you're thinking that you'll just use your calculator to do the math, don't forget that *calculators are not allowed on the SAT Subject Physics test*. Here is a summary of the really important math facts and formulas.

Exponents

$$x^a \cdot x^b = x^{a+b} \qquad x^a/x^b = x^{a-b} \qquad 1/x^b = x^{-b}$$

$$(x^a)^b = x^{a\cdot b} \qquad (xy)^a = x^a \cdot y^a \qquad (-1)^n = \begin{cases} +1, & \text{if } n \text{ is even;} \\ -1, & \text{if } n \text{ is odd.} \end{cases}$$

$$x^0 = 1 \qquad \sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$$

Scientific Notation

Scientific notation is a short-hand form to write numbers which would have a lot of zeros when written as decimals. For example, instead of writing 1230000, you can just write 1.23×1000000, or 1.23×10^6. The familiar powers of ten include:

$$10^{-3} = 0.001, 10^{-2} = 0.01, 10^{-1} = 0.1, 10^0 = 1, 10^1 = 10, 10^2 = 100, 10^3 = 1000.$$

To go from scientific notation to a plain decimal number, move the decimal to the right or left according to the sign of the exponent, putting a zero down when you have no other digits there. For example, for 3.7×10^{12}, move the decimal right 12 places and add 11 zeros. Move the decimal to the left for a negative exponent.

$$37\overbrace{00000000000}^{\text{11 zeros}}. = 3.7 \times 10^{12}$$

$$.\underbrace{0000000000}_{\text{10 zeros}}23 = 2.3 \times 10^{-11}$$

To go from a plain decimal number to scientific notation, just move the decimal to the right or left (counting how many places you move) until there is only one digit to the left of the decimal point, then add " $\times 10^n$ " where n is the number of places you moved the decimal point (positive if you went left and negative if you went right).

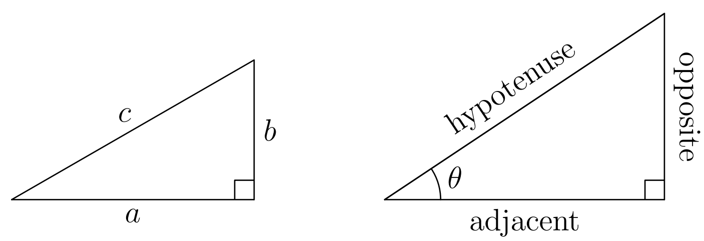
SAT Subject Physics Facts & Formulas

Basic Metric Prefixes

Common powers of ten (both positive and negative) have names that come before the metric unit of measurement, i.e., they are prefixes. The most typically used ones are given below.

Prefix	**Symbol**	**Power of Ten**	**Common Example**
nano	n	10^{-9}	nanometer
micro	μ	10^{-6}	microsecond
milli	m	10^{-3}	milligram
centi	c	10^{-2}	centimeter
kilo	k	10^{3}	kilogram
mega	M	10^{6}	megawatt

Basic Trigonometry



In the first triangle above,

$$a^2 + b^2 = c^2 \qquad \text{(pythagorean theorem)}$$

Referring to the second triangle, there are three important functions which are defined for angles in a right triangle:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

"SOH" "CAH" "TOA"

(the last line above shows a mnemonic to remember these functions: "SOH-CAH-TOA")

An important relationship to remember which works for any angle θ is:

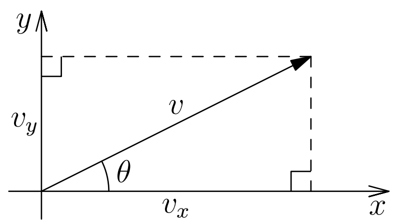
$$\sin^2\theta + \cos^2\theta = 1.$$

Vectors

Many important quantities in physics are represented by *vectors*, which specify both a number (the length of the vector) along with a direction (where the vector points). In contrast, *scalars* are simple numbers without a direction.

For example, velocity is a vector (represented by a boldface **v**) and is given by a number (say, 50 m/sec) along with a direction (say, 30° north of east). Mass (m) is just a number (say, 80 kg), for which a direction doesn't make any sense, so it is a scalar.

We can define *components* of a vector as the projection (or "shadow") of the vector on the x and y axes, as in the figure below.

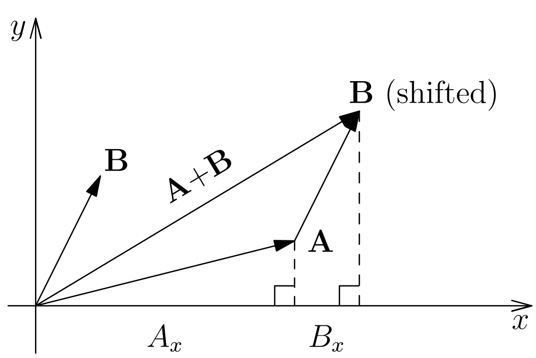


Using basic trigonometry,

$$v_x = v \cdot \cos\theta \quad \text{(the x-component of } \mathbf{v}\text{)}$$
$$v_y = v \cdot \sin\theta \quad \text{(the y-component of } \mathbf{v}\text{)}$$

Note from the figure that v (which is sometimes denoted explicitly by $|\mathbf{v}|$, which means the length of the vector **v**) is given by $v^2 = v_x^2 + v_y^2$, using the pythagorean theorem. In the example above, $v = 50$ m/sec and $\theta = 30°$, so that $v_x = 43$ m/sec and $v_y = 25$ m/sec. In this case, the x-component of **v** is greater than the y-component of **v** since the direction of **v** is closer to the x-axis (east) than it is to the y-axis (north).

The easiest way to add two vectors is to add their x components to get a total x component, and separately do the same thing for the y components. Then, a new total vector can be made with the two total x and y components, using $v_{\text{tot}}^2 = v_{\text{x,tot}}^2 + v_{\text{y,tot}}^2$ and $\theta = \tan^{-1}(v_{\text{y,tot}}/v_{\text{x,tot}})$. Graphically, this is the same as the "tip-to-tail" method, as in the figure below.



Here, vectors **A** and **B** are added by moving **B** so that its tail is at the tip of **A**, and then drawing the vector from the origin to the new tip of **B**. It should be clear from the figure that the x components of **A** and (the shifted) **B** add up to the x component of the new vector, and similarly for the y components.

SAT Subject Physics Facts & Formulas

Kinematics

The following formulas for position x, velocity v, and acceleration a are valid when the acceleration of the object is constant. The initial value of a variable, such as position for example, is given by x_i, and the final value is given by x_f. The change in the variable, such as velocity for example, is given by $\Delta v = v_f - v_i$.

There are five main equations for kinematics which are all valid, but the one or two that you use will depend on the variable that you need and the information that you have.

When you don't have	**Equation to Use**
a	$\Delta x = v_{\text{ave}}\Delta t = \frac{1}{2}(v_i + v_f)\Delta t$
Δx	$\Delta v = a\Delta t$
v_f	$\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$
v_i	$\Delta x = v_f\Delta t - \frac{1}{2}a(\Delta t)^2$
Δt	$v_f^2 = v_i^2 + 2a\Delta x$

A note about graphs: the slope of a position vs. time graph is the velocity. Also, the slope of the velocity vs. time graph is the acceleration.

Dynamics

Dynamics is the application of Newton's Laws to determine how a mass m moves when a force (or forces) is applied.

Newton's First Law is: *an object which moves at a constant velocity will continue moving at the same velocity unless it is acted upon by an non-zero force.* The force could be a single force, or several forces which are unbalanced (don't add to zero). Note that an object at rest has a constant velocity of zero, so it will remain at rest unless acted upon by such a force.

Newton's Second Law is: *the force on a mass equals the mass multiplied by the acceleration.* As a formula:

$$\mathbf{F} = m\mathbf{a}$$

where $\mathbf{F}$ is the force vector, m is the mass, and $\mathbf{a}$ is the acceleration vector. It is important to remember that the force in $F = ma$ is the sum of all the forces (often called the "net force") acting on the mass, not just one particular force. The net force acting on a book resting on a table is zero: the weight of the book and the force of the table pushing up on the book add to zero. Note that the weight of an object (which is a *force*) due to gravity is:

$$W = mg$$

where W is the weight, m is the mass, and g is the acceleration due to gravity (g is approximately 10 m/s^2 at or near the surface of the earth).

Friction is a force due to the contact of two rough surfaces against one another. The direction of the friction force is always opposite to the direction of motion (or, in the case of *static* friction, opposite to the motion that would occur if there were no friction). The magnitude of the friction force is proportional to the normal force holding the two surfaces together. The constant of proportionality is called the coefficient of friction, and is denoted by μ (this symbol is the Greek letter mu). In formula form:

$$f = \mu N$$

where f is the friction force, μ is the coefficient of friction, and N is the normal force. If there is no motion between the surfaces, the friction is static, and $\mu = \mu_s$, the static coefficient of friction. In the case of the two surfaces moving against one another, $\mu = \mu_k$, the kinetic coefficient of friction. Generally, μ_s is larger than μ_k; however, the basic formula $f = \mu N$ remains the same for both cases.

Momentum

Momentum is defined to be the product of mass and velocity:

$$\mathbf{p} = m\mathbf{v}$$

where $\mathbf{p}$ is the momentum, m is the mass, and $\mathbf{v}$ is the velocity. Note that the momentum $\mathbf{p}$ and velocity $\mathbf{v}$ are both vectors, and they are in the same direction, since the mass m is just a positive number.

The net force F acting on a mass m for an amount of time Δt produces a change in momentum given by

$$\Delta p = F\Delta t.$$

The product $F\Delta t$ is often called the *impulse*. Here, the change in momentum is just $\Delta p = m\Delta v = m(v_f - v_i)$, where v_i is the initial velocity and v_f is the final velocity.

Conservation of momentum: if there are no external forces on a system (or, the forces add to zero), then the momentum of a system is *conserved*, i.e., the momentum is constant. For example, consider when a rifle fires a bullet (in which case the "system" consists of the rifle plus the bullet). Before firing, $p = 0$. Since the external forces on the system add to zero (the weight of the rifle is balanced by the person holding it), then $p = 0$ after firing, also. Therefore, $m_b v_b + m_r v_r = 0$ and the recoil velocity of the rifle is $v_r = -(m_b/m_r) \cdot v_b$.

In collisions, momentum is also conserved. For example, suppose two cars (masses m_1 and m_2) collide at velocities v_1 and v_2. Then, their velocities after colliding (v_1' and v_2') satisfy the equation $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$. To solve a problem like this for the individual final velocities, more information must be given. For example, if the cars stick together, then the final velocity v_f can be found by solving $m_1v_1 + m_2v_2 = (m_1 + m_2) \cdot v_f$ for v_f.

Work, Energy, and Power

Work has a very specific meaning in physics. Work is done when a force is applied to an object as it moves a distance. The amount of work done is given by:

$$W = Fd\cos\theta$$

where W is the work done, in joules (1 J = 1 newton-meter), d is the distance, and θ is the angle between the direction of the force and the direction of motion. If the force is in direction of motion, $\theta = 0°$, so $\cos\theta = 1$ and the work is just force times distance, $W = Fd$.

If there is no displacement at all (i.e., $d = 0$), then no work is done. Also, if the force is at right angles to the direction of motion ($\theta = 90°$), again no work is done, since $\cos 90° = 0$. Note that work can be negative, for example, if $\theta = 180°$, then $W = -Fd$ and $W < 0$ ($\cos 180° = -1$). The work done by kinetic friction, for example, is always negative since the force of friction is always directed opposite to the direction of motion.

The *energy* of an object can be thought of as the ability of that object to do work. Or, conversely, work must be done by or on an object to change the object's energy. There are two main kinds of energy. The first is called *kinetic* energy and is associated with objects that are moving ($v \neq 0$). The amount of kinetic energy is given by

$$\text{KE} = \frac{1}{2}mv^2$$

where m is the mass and v is the velocity. For example, when someone throws a 0.5 kg softball, the softball goes from rest ($v = 0$) to some velocity (say, 8 m/s). The kinetic energy of the softball has increased from zero to $1/2 \cdot 0.5\text{ kg} \cdot (8\text{ m/s})^2 = 16$ joules, so the thrower has done +16 joules of work on the softball. When the ball is caught, the catcher must do (negative) work on the ball (namely, -16 J) to change its energy from 16 joules back to zero. Equivalently, the ball will do 16 joules of work on the catcher as it comes to a halt.

The second main kind of energy is called *potential* energy. This energy is associated with the position of the object (for example, the height of a mass measured from the ground below) or its configuration (for example, a compressed spring). In the case of a mass m at a height h, the potential energy is

$$\text{PE} = mgh.$$

Just as before, work must be done on or by the object to change its energy. In this case, perhaps the mass was carried by someone on the ground up to the height h, at a constant velocity. The work done by the person is the force applied (mg, to counteract gravity) times the distance (h), which gives $W = mgh$. This work is "stored" as potential energy in the mass at height h.

The mass now has the ability to do work. For example, if the mass is a large boulder at the top of a cliff, the boulder could be used to make a crater in the ground below (by pushing it off the cliff). The amount of work done by the boulder on the ground would be $W = mgh$.

We can combine the potential and kinetic energy to get the total energy:

$$E = \text{KE} + \text{PE}.$$

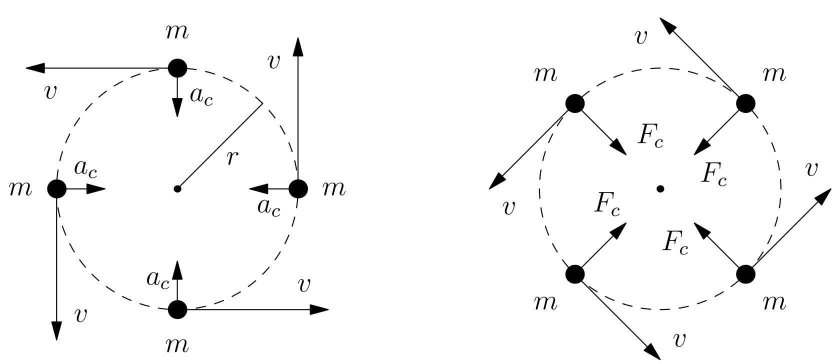
In the absence of forces such as friction, the total energy of an object remains constant (is *conserved*). For example, suppose the boulder from before is 100 kg and is on a cliff 10 m high. The total energy of the boulder is $E = \text{KE} + \text{PE} = 0 + (100\,\text{kg})(10\,\text{m/s}^2)(10\,\text{m}) = 10000\,\text{J}$. Now the boulder is pushed off the cliff. Just before it hits the ground, E is still 10000 J. But now, PE $= 0$ and KE $= 10000\,\text{J}$. Halfway down the cliff, PE $= (100\,\text{kg})(10\,\text{m/s}^2)(5\,\text{m}) = 5000\,\text{J}$. Since E is still 10000 J, then we know that KE $= 5000\,\text{J}$. In each case, the velocity could be found by using KE $= 0.5 \cdot m \cdot v^2$.

Power is the rate at which work is done: if a certain amount of work W is done in an elapsed time Δt, the power is

$$P = \frac{W}{\Delta t}.$$

For example, a 50-watt incandescent light bulb uses 50 joules of energy for each second that it is turned on. A 100-watt bulb uses that same amount of energy in only 0.5 seconds, since $P = 100\,\text{W} = 50\,\text{J}/0.5\,\text{sec}$. (In both cases, about 90% of the work being done by the light bulbs goes into heat, not light! Fluorescent and LED lights are much more efficient.)

Circular Motion



When an object moves in a circle with a constant speed, the object is said to be in *uniform circular motion*. In the figure above, an object of mass m is moving uniformly in a circle of radius r, counter-clockwise, and the position of the mass is shown at eight different points on the circle. This mass is under constant acceleration, not because the speed is changing (it isn't) but because the *direction* of the velocity is changing (remember that velocity is a vector, and so it has both a magnitude (speed) and a direction).

In the figure above, the force on the mass (F_c) along with its acceleration (a_c) are shown at eight snapshots in time during the motion of the mass around the circle. Both F_c and a_c always point directly toward the center of the circle, so in either circle in the above figure, a_c could be replaced by F_c, and vice versa.

The acceleration a_c, called "centripetal" acceleration, is given by

$$a_c = \frac{v^2}{r}.$$

The corresponding centripetal force, which keeps the mass m going around in the circle, is

$$F_c = \frac{mv^2}{r}.$$

The period T is the time it takes for the object to make one revolution. Given T, the velocity can be found using

$$v = \frac{2\pi r}{T}.$$

Here, $2\pi r$ is the distance the object goes in one revolution and T is how long it took to go that distance. Sometimes, instead of T, the frequency f is given, where f is the number of revolutions per second. These two numbers are related by

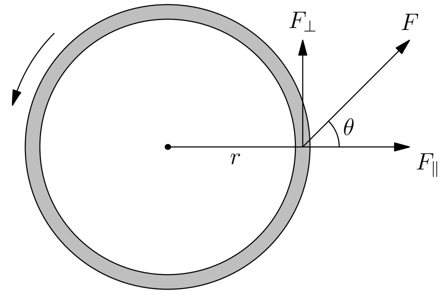
$$f = 1/T.$$

Torque And Angular Momentum

Torque is a force (F) that tends to make an object rotate. To do that, the force must act at a distance (r) from an axis of rotation:

$$\text{torque} = rF_\perp.$$

Here, $F_\perp$ is the component of the force F that is perpendicular to the direction of the radius. In the figure below, a force F is applied to an object (a wheel, say). The component of the force F that is parallel to the radius ($F_\parallel$) can't make the wheel rotate, and so it doesn't contribute to the torque.



If θ is the angle between the radial direction and F, then $F_\perp = F \sin\theta$, so that the torque equation can also be written as: torque $= rF \sin\theta$. The perpendicular force $F_\perp$ will tend to rotate the wheel in the figure in the counter-clockwise direction.

Angular momentum is the circular equivalent of linear momentum ($p = mv$), and is given by:

$$L = mvr.$$

For example, $L = mvr$ is the angular momentum of the rotating object in the diagram at the beginning of the section on circular motion.

Similarly to regular (linear) momentum, if there are no external torques on a system (or, the torques add to zero), then the angular momentum of a system is *conserved*, i.e., the angular momentum is constant. For example, could the Earth just stop spinning in the middle of the night? The biggest force on the Earth is due to the Sun (see the Gravity section below), but that force effectively acts at the center of the Earth, where $r = 0$. This means that the torque due to the force is zero, so there are no external torques on the Earth, and therefore it will just keep on spinning.

A little more complicated example is the system of the Earth and the Moon. The Moon (which has a large mass) moves roughly in a circle about the Earth, and so it has angular momentum. The Earth has angular momentum since it is spinning about its own axis once per day (and this includes *you*, since you are on the surface of the Earth, rotating about the axis). The Moon causes the tides on the Earth, but the drag of the sea sloshing about is gradually slowing down the spin rate of the Earth. The conservation of angular momentum in the Earth-Moon system implies that the angular momentum of the moon must be increasing, namely, the r in the angular momentum (mvr) of the Moon must be getting bigger. In fact, the distance to the moon has been measured to be increasing at roughly 4.5 cm per year.

Springs

A spring is a metal coil which, when stretched, pulls back on the object attached to the end of the spring. When compressed, the spring pushes against the object at the end of the spring. When not stretched or compressed, the spring is at its "natural length" and it doesn't exert a force on the object at all.

The restoring force F_s of a spring is proportional to the amount (distance) that the spring is stretched or compressed. If this distance is x, then the restoring force is

$$F_s = kx.$$

The formula above is often called "Hooke's Law". When a spring is stretched or compressed, it has a (stored) potential energy of

$$\text{PE}_s = \frac{1}{2}kx^2.$$

Gravity

Gravity is a force that occurs between objects that have mass. If two masses m_1 and m_2 are separated by a distance r, then the force of gravity between them is

$$F_g = G\frac{m_1 m_2}{r^2}$$

where G is just a number which is always the same for every calculation, i.e., G is a constant. In metric units, the number turns out to be $6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$, but it isn't important to know this particular number.

The above mass pairs could be everything from two billiard balls (same mass) to you (very small mass) and the earth (very large mass). Notice that the force of gravity is inversely proportional to the distance of separation, and proportional to the product of the two masses. For example, if the distance between m_1 and m_2 were to double, then the force would be only 25% as large. If the mass of the earth were doubled, the force on you (for example) would become twice as big, i.e., you would weigh twice as much as you do now!

Still under construction. More stuff (E&M, etc.) coming soon!